Gravitational collapse and evolution of holographic black holes

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Abstract. Gravitational collapse is analyzed in the Brane-World by arguing that regularity of five-dimensional geodesics require that stars on the brane have an atmosphere. For the simple case of a spherically symmetric cloud of non-dissipating dust, conditions are found for which the collapsing star evaporates and approaches the Hawking behavior as the (apparent) horizon is being formed. The effective energy of the star vanishes at a finite radius and the star afterwards re-expands and "anti-evaporates". Israel junction conditions across the brane (holographically related to the matter trace anomaly) and the projection of the Weyl tensor on the brane (holographically interpreted as the quantum back-reaction on the brane metric) contribute to the total energy as, respectively, an "anti-evaporation" and an "evaporation" term.

1. Introduction

Black holes (BHs) are unstable in four (and higher) dimensions because of the Hawking effect [1], which is deeply linked to the trace anomaly of radiation fields [2]. In the Randall-Sundrum (RS) Brane-World (BW) models [3], a collapsing homogeneous star likewise requires a non-static exterior [4]. Further, forcing a static exterior induces a trace anomaly of the same form as that of semiclassical BHs, although with opposite sign, which suggested that BH solutions of the bulk equations correspond to quantum corrected (semiclassical) BHs on the brane [5, 6], in the spirit of the holographic principle [7] and AdS/CFT conjecture [8].

The junction conditions [9], which preserve the regularity of (five-dimensional) geodesics, cannot allow a step-like discontinuity (e.g. across the star surface) and a δ-like discontinuity (e.g. across the brane) in the stress tensor at the same location, hence discontinuities in the stress tensor of brane stars are not mathematically permitted. This can be physically understood by considering that the brane thickness (of the order of the AdS length $\ell \sim \lambda^{-1/2}$, λ being the brane tension) and the star's atmosphere cannot be both negligibly thin, and it is indeed natural to assume that the latter is much larger than ℓ. In Ref. [10], we employed effective four-dimensional (hydrodynamical) equations [11] in order to study a "corrected" Oppenheimer-Snyder (OS) model [12] in which the star is divided into three regions (see Fig. 1): a homogeneous "core" with most of the energy; a "transition region" of fast density decrease which connects with a "tail", where the energy density slowly vanishes. The tail and transition region together form the "BW atmosphere", which disappears in the limit $\lambda \to \infty$.

In general, effective equations cannot determine the brane metric uniquely unless one also knows the bulk geometry. However, for negligible dissipation and asymptotically flat brane, the evolution of the system is uniquely determined by the dynamics of the homogenous core, which corresponds to an exact five-dimensional solution [14] and reproduces the trace anomaly of quantum field theory [2]. Our main results are that *the total energy of the system is conserved* and that *the collapsing star "evaporates" until the core experiences a "rebound" in the high energy regime (when its energy density is comparable with λ), after which the whole system "anti-evaporates"*. This behaviour cannot be related to GR perturbatively (in $\epsilon \equiv \rho_0/\lambda$, where ρ_0 is the initial core density), but it seems in agreement with the uncertainty principle of quantum mechanics [15]. Moreover, we can find a range of parameters for which the minimum radius of the collapsing core is larger than ℓ (which sets the scale of Quantum Gravity in the BW), thus supporting the holographic interpretation of the model.

2. Spherically symmetric collapsing dust

Effective four-dimensional Einstein equations for dust and vanishing brane cosmological constant can be written as [11] $G_{\mu\nu} = 8\,\pi\,T^{\rm eff}_{\mu\nu} = 8\,\pi\,\left(\rho^{\rm eff}\,u_\mu\,u_\nu + p^{\rm eff}\,h_{\mu\nu} + \Pi_{\mu\nu}\right)$, where u^μ is the unit four-velocity of dust, $h_{\mu\nu}$ the space-like metric that projects orthogonally to u^μ,

$$\rho^{\rm eff} = \rho\,\left(1+\frac{\rho}{2\,\lambda}\right) + \mathcal{U}\ , \quad p^{\rm eff} = \frac{\rho^2}{2\,\lambda} + \frac{\mathcal{U}}{3}\ , \tag{1}$$

with ρ the ("bare") energy density, and $\mathcal{U}$ and $\Pi_{\mu\nu}$ come from the Weyl tensor projected on the brane [1]. Bianchi identities supplied by the junction conditions then produce both local (LCE) and non-local conservation equations (NLCE). For negligible dissipation, one can take a Tolman brane geometry, $\mathrm{d}s^2 = -\mathrm{d}\tau^2 + (R')^2\,\mathrm{d}r^2 + R^2\,\mathrm{d}\Omega^2$ where $R = R(\tau, r)$, and the LCEs imply conservation of the "bare" mass function

$$m_\rho(r) \equiv \frac{4\,\pi}{3}\int_0^r \rho(\tau,x)\,\partial_x\left(R^3(\tau,x)\right)\,\mathrm{d}x\ . \tag{2}$$

The system of NLCEs is in general not closed, since we do not have an equation for $\Pi_{\mu\nu}$. However, for sufficiently large R, the knowledge of $\Pi_{\mu\nu}$ in an extended spatial region together with the asymptotic flatness and the continuity of the Weyl tensor make that system closed. Another important result which follows from the LCEs and NLCEs is that, *if the brane metric is asymptotically flat, the anisotropic stress* $\Pi_{\mu\nu} \neq 0$ *whenever* $\dot{\rho}' \neq 0$.

The (τ,τ) Einstein equation yields the equation of motion

$$\dot{R}^2(\tau,r) = \frac{2\,M(\tau,r)}{R(\tau,r)}\ . \tag{3}$$

where the (in general time-dependent) "effective" mass

$$M(\tau,r) = \frac{4\,\pi}{3}\int_0^r \rho^{\rm eff}(\tau,x)\,\partial_x\left(R^3(\tau,x)\right)\,\mathrm{d}x\ , \tag{4}$$

replaces the bare mass of General Relativity (GR). The latter is always well defined and dust shells in GR move along geodesics to reach the central singularity ($R = 0$) at increasing proper times (Tolman model [13]) or at the same proper time (OS model [12]). In the BW, the effective mass M drives the collapse, but it diverges for $R \to 0$ and this would make the whole four-dimensional brane singular. In order to avoid this (physically unlikely) case, one has to include a sufficiently negative contribution to the mass from the projected Weyl tensor which will generate an Hawking flux near the forming horizon and make the effective mass evaporate completely.

[1] We set the momentum density $Q_\mu = 0$, see Ref. [10] for more details.

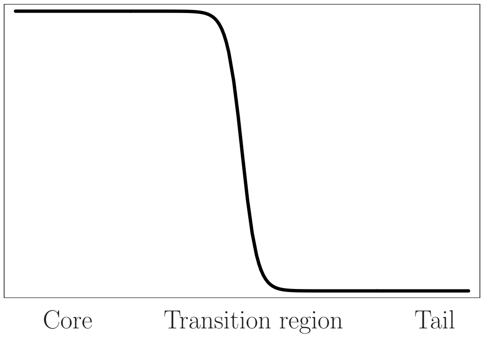

Figure 1. Density profile.

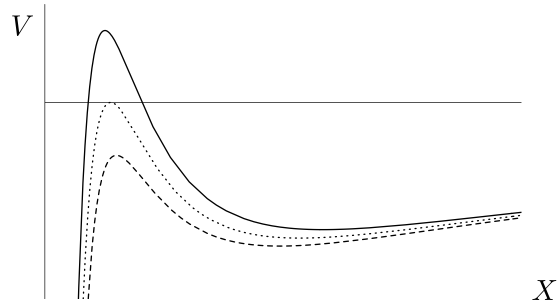

Figure 2. Qualitative behavior of V for $\mu > \mu_{\rm c}$ (solid line) and $\mu < \mu_{\rm c}$ (dashed line). For $\mu = \mu_{\rm c}$ (dotted line), the peak of V equals the shells energy $E = 0$.

2.1. Core

The bulk solution which corresponds to the OS core of the star is regular in five dimensions [14] and, since $\rho' = 0$, the system of relevant equations is now closed with $\Pi = 0$ and

$$\mathcal{U} = -\frac{27\,\mu\,r^4\,\epsilon}{128\,\pi^2\,r_0^4\,\rho_0\,R^4}\ , \tag{5}$$

where μ is a constant. The physical radius R can be written in the factorized form

$$R(\tau, r) = \left(\frac{9}{2}\,M_{\rm S}\right)^{1/3}\frac{r}{r_0}\,X(\tau)\ , \tag{6}$$

in which $M_{\rm S}$ is the total bare mass of the OS core, and the effective mass (4) is given by

$$M(\tau, r) = M_{\rm S}\left(\frac{r}{r_0}\right)^3 + \frac{9\,\epsilon}{32\,\pi\,\rho_0}\left(\frac{r}{r_0}\right)^4\left[\frac{(2\,M_{\rm S})^2}{3\,R^3}\left(\frac{r}{r_0}\right)^2 - \frac{\mu}{R}\right]\ , \tag{7}$$

where the second term represents a BW correction. The above mass would diverge for $R \to 0$, the harmless (at least when covered by an horizon) central singularity of GR, and make the whole space-time singular (this also occurs for a more general Tolman core [10]). One must therefore have μ positive and large enough so that each shell will bounce back after reaching a minimum radius $R_{\rm min}$ where the corresponding effective mass vanishes. The Weyl tensor, holographically interpreted as the quantum back-reaction on the brane metric, contributes the "evaporation" term proportional to μ which dominates at low energies; the BW correction to the matter stress tensor, holographically interpreted as the matter quantum trace anomaly [5], yields the "anti-evaporating" term proportional to M_S^2 which increases with the energy.

Upon inserting the effective mass (7) into Eq. (3), one obtains

$$\dot{X}^2 = \frac{4}{9\,X} + \frac{\epsilon}{27\,\pi\,\rho_0\,X^4} - \frac{6^{1/3}\,\epsilon\,\mu}{24\,\pi\,\rho_0\,M_{\rm S}^{4/3}\,X^2} \equiv -V(X)\ , \tag{8}$$

which shows that the core remains "rigid" through the bounce and it is thus sufficient to consider the evolution of its surface at $r = r_0$. There exists a critical value $\mu_{\rm c} = \left(32\,\pi\,\rho_0\,M_{\rm S}^2/3\,\epsilon\right)^{2/3}$ such that the bounce occurs only for $\mu > \mu_{\rm c}$ (for $\mu = \mu_{\rm c}$ the two turning points coincide, see Fig. 2). In Fig. 3 we display a typical trajectory of $R_0(\tau) = R(\tau, r_0)$, along with the corresponding effective

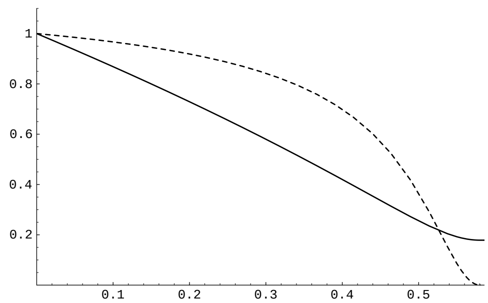

Figure 3. Typical evolutions of the core radius $R_0(\tau)/R_0(0)$ (solid line) and effective mass $M_0(\tau)/M_0(0)$ (dashed line) for $\mu > \mu_c$. Time is arbitrary.

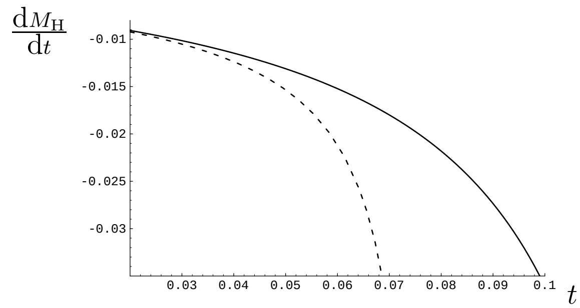

Figure 4. Energy flux versus the time $t = (\tau - \tau_H^{OS})/T$ (solid line) compared to the Hawking flux (dotted line) for $\epsilon = 10^{-4}$, $M_S = \rho_0 = 1$, $T = 10$ and $\mu = 5000 > 4824 = \mu_c$.

mass $M_0(\tau) = M(\tau, r_0)$, for $\mu > \mu_c$. After the rebound, the system reverses its evolution, however, in a more realistic model, the collisionless description of dust should be relaxed and dissipation from both the core and the atmosphere is expected to make the process irreversible.

Since $\dot{R}_0 < 0$ and $R_0(0) \gg 2\,M_S$ (there is no initial horizon), for $\mu > \mu_c$

$$\dot{M}_0(\tau) = -\frac{9\,\epsilon}{32\,\pi\,\rho_0}\left[\left(\frac{2\,M_S}{R_0}\right)^2 - \mu\right]\frac{\dot{R}_0}{R_0^2} < 0 \ , \tag{9}$$

and the evaporation sets out at the beginning of the collapse and goes on at least until the radius bounces back at $R_{min} \sim \ell\,(M_S/\ell)^{1/3} \gg \ell \sim \lambda^{-1/2}$. Since R_{min} is the shortest length in our system, the holographic description is expected to hold for astrophysical sources for which $M_S \gg \lambda^{-1/2}$ (it also follows that $\mu_c \gg 1$).

The horizon trajectory $r = r_H(\tau)$ is defined as the value of r at which the horizon is formed at the time τ, that is $R(\tau, r_H(\tau)) = 2\,M(\tau, r_H(\tau))$. The time derivative of the horizon effective mass, $M_H(\tau) \equiv M(\tau, r_H(\tau))$,

$$\frac{dM_H}{d\tau} = \dot{M}_H + M_H'\,\dot{r}_H \ , \tag{10}$$

then contains two contributions: one, from the intrinsic time dependence of M at constant r, is a first order effect in ϵ which vanishes in GR; the second one, due to the (possibly) variable number of shells included within the horizon, depends on the detailed form of the atmosphere.

Since in the core, the velocity $|\dot{R}(\tau, r)|$ increases monotonically in r at fixed τ, an (apparent) horizon forms at the boundary $r = r_0$ at $\tau = \tau_H^{OS} \equiv T - \frac{4}{3}\,M_S$, where T fixes the time scale of the collapse. From Eq. (9), we then get the (instantaneous) Hawking flux [1]

$$\frac{dM_H}{d\tau} \simeq -\frac{9\,(\mu - 1)\,\epsilon}{128\,\pi\,\rho_0\,M_H^2} \ , \tag{11}$$

precisely at the time $\tau = \tau_H^{OS}$.

2.2. Transition region

In the GR model $\rho = 0$ for $r > r_0$. The density must therefore decrease rapidly from $\rho = O(\epsilon^0)$ at $r = r_0$ to $\rho = O(\epsilon)$ for $r = r_s$. Moreover, since the transition is a BW effect, we can take $r_s - r_0 = O(\epsilon)$ as well as $\mathcal{U} = O(\epsilon)$. Combining these results, we obtain, to first order in ϵ,

$$\dot{M}(\tau, r) \simeq \dot{M}_0(\tau) \ , \tag{12}$$

for $r_0 < r < r_s$, and the Hawking flux will remain negative (and substantially unaffected) throughout the border of the transition region $r = r_s$.

2.3. Tail

As in the transition region, $\rho = O(\epsilon)$ for $r_s < r$, and $\rho'\,\rho/\lambda = O\left(\epsilon^2\right)$, so that bulk gravitons are decoupled from brane matter. The Weyl contribution is however of the same order, $m_{\mathcal{U}}(\tau, r; r_s) = \frac{4\pi}{3}\int_{r_s}^{r} \mathcal{U}\, \left(R^3\right)'\, \mathrm{d}x = O(\epsilon)$, and satisfies

$$\dot{m}_{\mathcal{U}} + 2\sqrt{\frac{2\,M_{\mathrm{S}}}{R^3}}\, m_{\mathcal{U}} = \dot{M}_0\, \left[\left(\frac{R_0}{R}\right)^{3/2} - 1\right] . \tag{13}$$

Since $R(\tau, r) > R_0(\tau)$ for $r > r_0$, (13) implies that $m_{\mathcal{U}}$ cannot remain zero in the tail.

In order to proceed, we now assume that:

(i) $\lim_{r\to\infty} R(\tau, r) = \infty$, and

(ii) *the effective mass be finite at spatial infinity*, $\lim_{r\to\infty} M(\tau, r) < \infty$, $\forall\, \tau > 0$.

Since $M_0(\tau)$ always remains finite if $\mu > \mu_c$ and the tail bare mass is small by construction, this implies that $\lim_{r\to\infty} m_{\mathcal{U}}(\tau, r; r_s) < \infty$, $\forall\, \tau > 0$. Asymptotic flatness ensures that we can take the limit $r \to \infty$ (equivalent to $R \to \infty$ at fixed time) in Eq. (13) and finally obtain

$$\lim_{r\to\infty} \dot{m}_{\mathcal{U}}(\tau, r; r_s) + \dot{M}_0(\tau) = \lim_{r\to\infty} \dot{M}(\tau, r) = 0\ , \quad \forall\, \tau > 0\ . \tag{14}$$

We have thus shown that *if the total effective mass at spatial infinity is finite at the initial time* $\tau = 0$*, it will always remain constant* (for a bouncing core evolution with $\mu > \mu_c$), so that *the total effective mass of the collapsing dust star is actually conserved.*

It is particularly interesting to consider the case for which there is initially no energy stored in $\mathcal{U}$. We then see from Eq. (13) that $m_{\mathcal{U}} > 0$ during the collapse and, after the bounce, we expect that $m_{\mathcal{U}}$ will also evolve backwards so as to ensure the condition (14). The energy flux along the horizon trajectory $r = r_{\mathrm{H}}(\tau)$ can also be obtained numerically for this case and is plotted in Fig. 4 for some values of the parameters. We can see that it grows less with respect to that predicted by Hawking and the conclusion is that, although the evaporation sets out according to Hawking's law, the back-reaction on the brane metric subsequently reduces the emission.

3. Luminosity

A distant observer experiences an impinging flux of energy during the collapse,

$$\Phi_t \equiv \frac{\mathrm{d}}{\mathrm{d}\tau}\left[\lim_{\bar{r}\to\infty} \frac{4\,\pi}{3} \int_{r_{\mathrm{H}}(\tau)}^{\bar{r}} \rho^{\mathrm{eff}}\, \left(R^3\right)'\, \mathrm{d}r\right] \simeq -\frac{\mathrm{d}M_{\mathrm{H}}}{\mathrm{d}\tau}\ , \tag{15}$$

which shows the same dependence on the mass M_{H} as the semiclassical expression when the horizon is first forming, and subsequently decreases to zero (before it becomes negative). However, since this happens after the apparent horizon begins to form, a distant observer might have to wait an infinite amount of time to measure a vanishing flux.

Upon equating the BW result (11) at the time when the horizon starts to form to the semiclassical luminosity calculated in the Schwarzschild background for an astrophysical object, we obtain a bound for the AdS length [10],

$$10^{-32}\,\mathrm{mm} \ll \ell \ll 10^{-9}\,\mathrm{mm}\ , \tag{16}$$

which is three orders of magnitude better than the best constraint found in [16] considering the time scale of primordial BH evaporation.

4. Trace anomaly

Strictly speaking, there is no trace anomaly in our approach, since we have included the back-reaction of the effective matter on the brane metric. However, in order to compare with known results *without* the back-reaction, we can define the trace anomaly $\mathcal{R}$ as the sum of the Ricci scalar and the trace of the *bare* stress tensor. At the OS boundary, $r = r_0$, we then have

$$\mathcal{R} = -\frac{9}{2\,\pi\,\lambda}\,\frac{M_{\rm S}^2}{R^6}\ , \tag{17}$$

which is the quantum Ricci anomaly of Ref. [2] with the correct sign. It is then clear that the sign mismatch found in Ref. [4] was due to the choice of a non-smooth energy density and that the atmosphere describes how the excess energy stored in the OS boundary is released.

5. Conclusions

Inspired by the conjecture that classical BHs in the BW may reproduce the semiclassical behavior of four-dimensional BHs, we have studied the gravitational collapse of a spherical star of dust in the RS scenario. Regularity of the bulk geometry requires continuity of the matter stress tensor on the brane and leads to a loss of mass from the boundary of the star. We found that the system of effective BW equations is closed to our level of approximation and leads to the collapsing dust star emitting a flux of energy which, at relatively low energies, approaches the Hawking behavior when the (apparent) horizon is being formed. However, no real space-time singularity forms since the star effective mass vanishes at finite star radius, thus leaving a remnant which re-expands by absorbing back the emitted radiation. In a more realistic case, one however expects that dissipation cannot be neglected and the process then becomes irreversible.

Let us finally point out that all the above features were obtained for BHs formed by gravitational collapse, excluding therefore primordial BHs.

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